

# Distance Two Labeling of Triangular Snake Families

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**ABSTRACT:** An  $L(2, 1)$  labeling (or) distance two labeling of a graph  $G$  is a function  $f$  from the vertex set  $V(G)$  to the set of all non-negative integers such that  $|f(x) - f(y)| \geq 2$  if  $d(x, y) = 1$  and  $|f(x) - f(y)| \geq 1$  if  $d(x, y) = 2$ . The  $L(2, 1)$  labeling number  $\lambda(G)$  of  $G$  is the smallest number  $k$  such that  $G$  has an  $L(2, 1)$  labeling with  $\max\{f(v), v \in V(G)\} = k$ . In this paper the  $L(2, 1)$  labeling number  $\lambda(G)$  for some graphs like triangular snake, double triangular snake, alternate triangular snake, alternate double triangular snake, triple triangular snake and alternate triple triangular snake have been determined.

**KEYWORDS:**  $L(2, 1)$  labeling,  $\lambda$  number, cycle, path, triangular snake, double triangular snake, alternate triangular snake, alternate double triangular snake, triple triangular snake and alternate triple triangular snake

## I. INTRODUCTION

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Labeled graphs serve as useful models for broad range of applications such as coding theory, X-ray crystallography, radar, astronomy, circuit design, communication networks and data base management and models for constraint programming over finite domain. The concept of graph labeling was introduced by Rosa in 1967. Hence in the intervening years various labeling of graphs such as graceful labeling, harmonious labeling, magic labeling, antimagic labeling, bimagic labeling, prime labeling, cordial labeling, total cordial labeling,  $k$ -graceful labeling and odd graceful labeling etc., have been studied in over 2100 papers.

Hale [6] introduced the graph theory model of assignment of channels in 1980 in which a graph was considered whose vertices represented the stations and the edges represented the stations. The graphs considered here are all finite, undirected and simple.  $V(G)$  and  $E(G)$  denote the vertex set and the edge set of  $G$ . Graph labeling is an assignment of integers to vertices or edges or both under certain conditions.

Labeling with a condition of distance two was introduced by J.R. Griggs and R.K. Yeh [5] who proved that every graph with maximum degree  $k$  has an  $L(2, 1)$  labeling with span at most  $k+2k$  and proved the conjecture for 2-regular graphs. G.J. Chang and D. Kuo [1] improved this upper bound to  $k+2k$ . Chang et al. [2] generalized this to obtain  $k+2(d-1)k$  as an upper bound on the minimum span of an  $L(d, 1)$ -labeling. Z. Fredi and J.H. Kang [3] proved it for 3-regular Hamiltonian graphs and for the incidence graphs of projective planes. In this paper, we study the  $L(2, 1)$  labeling of triangular snake, double triangular snake, alternate triangular snake, alternate double triangular snake, triple triangular snake and alternate triple triangular snake.

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## II. PRELIMINARIES

Definition 2.1

An  $L(2, 1)$  labeling of a graph is a labeling of the vertices with non-negative integers such that the labels on adjacent vertices differ by at least 2 and the labels on vertices at distance 2 differ by at least 1. The  $L(2, 1)$  labeling number  $\lambda(G)$  of  $G$  is the smallest number  $k$  such that  $G$  has an  $L(2, 1)$  labeling with  $\max \{f(v), v \in V(G)\} = k$ .

Definition 2.2.

A triangular snake  $T_n$  is obtained from a path  $a_1, a_2, \dots, a_n$  by joining  $a_i$  and  $a_{i+1}$  to a new vertex  $b_i$  for  $i=1, 2, \dots, n-1$ . That is every edge of a path is replaced by a triangle  $C_3$ .

Definition 2.3.

An alternate triangular snake  $A(T_n)$  is obtained from a path  $a_1, a_2, \dots, a_n$  by joining  $a_i$  and  $a_{i+1}$  to a new vertex  $b_i$  for  $i \equiv 1 \pmod{2}$  and  $i \leq n-1$ . That is every alternative edge of a path is replaced by  $C_3$ .

Definition 2.4.

A double triangular snake  $D(T_n)$  is obtained from two triangular snakes with a common path.

Definition 2.5.

An alternate double triangular snake  $A(D(T_4))$  is obtained from two alternate triangular snakes with a common path.

Definition 2.6.

A triple triangular snake  $T(T_n)$  is obtained from a path  $u_1, u_2, \dots, u_n$  by joining  $u_i$  and

$u_{i+1}$  to three new vertices  $v_i, w_i$  and  $t_i$  for  $1 \leq i \leq n-1$ . That is, a triple triangular snake  $T(T_n)$  consists of three triangular snakes that have a common path.

Definition 2.7.

An alternate triple triangular snake  $A[T(T_n)]$  consists of three alternate triangular snakes that have a common path.

## III. MAIN RESULTS

Theorem 3.1.

The  $L(2,1)$  labeling number  $\lambda(G)$  of a of a triangular snake  $T_n$  is 7, for  $n \geq 3$ .

Proof.

Denote the vertices of a triangular snake graph as follows:

$V(G) = V_1 \cup V_2$  where  $V_1 = \{v_i / 1 \leq i \leq n-1\}$  and  $V_2 = \{w_i / 1 \leq i \leq n\}$

Define a mapping  $f: V(G) \rightarrow N \cup \{0\}$  by

$$f(v_{3i-2}) = 5; \quad 1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor$$

$$f(v_{3i-1}) = 6; \quad 1 \leq i \leq \left\lfloor \frac{n-1}{3} \right\rfloor$$

$$f(v_{3i}) = 7; \quad 1 \leq i \leq \left\lfloor \frac{n-3}{3} \right\rfloor$$

$$f(w_{3i-2}) = 0; \quad 1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor$$

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$$f(w_{3i-1})=2; \quad 1 \leq i \leq \left\lceil \frac{n-1}{3} \right\rceil$$

$$f(w_{3i})=4; \quad 1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor$$

Now let us find the L(2,1) labeling number of this graph.

Case(i):

Let x,y be any two vertices in  $V_1(G)$ .

Clearly x and y are non-adjacent vertices on  $V_1(G)$ , such that  $x = v_{3i-2}$  and  $y = v_{3i-1}$ .

Then  $f(x) = 5$ ,  $f(y) = 6$  and  $d(x, y) = 2$ .

Therefore  $d(x, y) + |f(x) - f(y)| = 2 + |5-6| = 3$ .

Case(ii):

Let x,y be any two vertices in  $V_2(G)$ .

Subcase (i).

Let x,y be any two adjacent vertices on  $V_2(G)$ , such that  $x = w_{3i-2}$  and  $y = w_{3i-1}$ .

Then  $f(x) = 0$ ,  $f(y) = 2$  and  $d(x, y) = 1$ .

Therefore  $d(x, y) + |f(x) - f(y)| = 1 + |0-2| = 3$ .

Subcase (ii).

Let x,y be any two non-adjacent vertices on  $V_2(G)$ , such that  $x = w_{3i-2}$  and  $y = w_{3i}$ . Then  $f(x) = 0$ ,  $f(y) = 4$  and  $d(x, y) \geq 2$ . Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |0-4| \geq 3$ .

Case(iii):

Let x,y be any two vertices, on the  $V_1(G)$  and  $V_2(G)$  respectively

Subcase (i).

Let x,y be any two adjacent vertices, on  $V_1(G)$  and  $V_2(G)$  respectively. Let x be a vertex on  $V_1(G)$  and y be a vertex on  $V_2(G)$  such that  $x = v_{3i-2}$  and  $y = w_{3i-2}$ . Then  $f(x) = 5$  and  $f(y) = 0$  and  $d(x, y) \geq 1$ . Therefore  $d(x, y) + |f(x) - f(y)| \geq 1 + |5-0| \geq 3$ .

Subcase (ii).

Let x,y be any two non-adjacent vertices, on  $V_1(G)$  and  $V_2(G)$  respectively.

Let x be a vertex on  $V_1(G)$  and y be a vertex on  $V_2(G)$  such that  $x = v_{3i-2}$  and  $y = w_{3i}$ . Then  $f(x) = 5$  and  $f(y) = 4$ , then  $d(x, y) \geq 2$ . Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |5-4| \geq 3$ .

Similarly, for all the other possibilities of x and y we find that  $d(x, y) + |f(x) - f(y)| \geq 3$ . Therefore the L(2, 1) labeling number of  $T_n$  is  $\lambda(G) = 7$ .

Example 3.1

The L(2,1) labeling of a triangular snake  $T_5$  is shown in Figure 3.1.

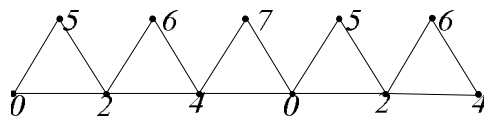


Figure 3.1

Hence the L(2,1) labeling of a triangular snake  $T_n$  is 7.

Theorem 3.2.

The L(2, 1) labeling number  $\lambda(G)$  of an alternate triangular snake graph  $A(T_n)$  is 6, for  $n \geq 2$  and  $n \equiv 0 \pmod{2}$

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Proof.

Denote the vertices of an alternate triangular snake graph  $A(T_n)$  as follows:

$V(A(T_n)) = V_1 \cup V_2$  where

$$V_1 = \{v_i / 1 \leq i \leq \frac{n}{2}\}$$

$$V_2 = \{w_i / 1 \leq i \leq n\}$$

Define a mapping  $f: V(G) \rightarrow \mathbb{N} \cup \{0\}$  such that

$$f(v_i) = 0, \text{ for } 1 \leq i \leq \frac{n}{2}$$

$$f(w_{3i-2}) = 2, \text{ for } 1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor$$

$$f(w_{3i-1}) = 4, \text{ for } 1 \leq i \leq \left\lfloor \frac{n-1}{3} \right\rfloor$$

$$f(w_{3i}) = 6, \text{ for } 1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor$$

Case(i):

Let  $s, t$  be any two vertices in  $V_1(G)$ . Clearly  $d(s, t) \geq 3$ . Hence the  $L(2,1)$  labeling condition is satisfied.

Case(ii):

Let  $s, t$  be any two vertices in  $V_2(G)$ .

Subcase(i).

Let  $s, t$  be any two adjacent vertices in  $V_2(G)$

such that  $s = w_{3i}$  and  $t = w_{3i-1}$ . Then  $f(s) = 6, f(t) = 4$  and  $d(s, t) = 1$ . Therefore  $d(s, t) + |f(s) - f(t)| = 1 + |6-4| \geq 3$ .

Subcase(ii).

Let  $s, t$  be any two non-adjacent vertices in  $V_2(G)$  such that  $s = w_{3i-2}$  and  $t = w_{3i}$  then  $d(s, t) \geq 2$ . Then  $f(s) = 2, f(t) = 6$  and  $d(s, t) = 2$ . Therefore  $d(s, t) + |f(s) - f(t)| \geq 2 + |2-6| \geq 3$ .

Case(iii):

Let  $s, t$  be any two vertices in  $V_1(G)$  and  $V_2(G)$  respectively

Subcase(i).

Let  $s, t$  be any two adjacent vertices in  $V_1(G)$  and  $V_2(G)$  respectively such that  $s = v_i$  and  $t = w_{3i-2}$  then  $f(s) = 0, f(t) = 2$  and  $d(s, t) = 1$ . Therefore  $d(s, t) + |f(s) - f(t)| = 1 + |0-2| \geq 3$ .

Subcase(ii).

Let  $s, t$  be any two non-adjacent vertices in  $V_1(G)$  and  $V_2(G)$  respectively such that  $s = v_i$  and  $t = w_{3i}$ ,  $f(s) = 0, f(t) = 6$  then  $d(s, t) \geq 2$ . Therefore  $d(s, t) + |f(s) - f(t)| \geq 2 + |0-6| \geq 3$ . Therefore the  $L(2,1)$  labeling condition is satisfied.

Similarly, for all the possibilities of  $s$  and  $t$  we find  $d(s, t) + |f(s) - f(t)| \geq 3$ . Therefore the  $L(2, 1)$  labeling number of  $A(T_n)$  is  $\lambda(G) = 6$

Example 3.2

$L(2,1)$  labeling of alternate triangular snake graph  $A(T_5)$  is shown in Figure 3,2

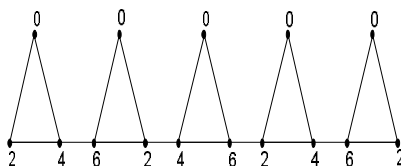


Figure 3,2

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Hence the  $L(2, 1)$  labeling number  $\lambda(G)$  of alternate triangular snake graph  $A(T_n)$  is 6.

Theorem 3.3

The  $L(2, 1)$  labeling number  $\lambda(G)$  of a double triangular snake  $D(T_n)$  is 9, for  $n \geq 3$ .

Proof.

Denote the vertices of a double triangular snake graph as follows:

$V(G) = V_1 \cup V_2 \cup V_3$  where  $V_1 = \{v_i / 1 \leq i \leq n-1\}$ ,  $V_2 = \{w_i / 1 \leq i \leq n\}$ ,  $V_3 = \{u_i / 1 \leq i \leq n-1\}$

Define a mapping  $f : V(G) \rightarrow \mathbb{N} \cup \{0\}$  by

$$f(v_{2i-1}) = 6; \quad 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$$

$$f(v_{2i}) = 7; \quad 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$$

$$f(w_{3i-2}) = 0; \quad 1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor \quad f(w_{3i-1}) = 2; \quad 1 \leq i \leq \left\lfloor \frac{n-1}{3} \right\rfloor$$

$$f(w_{3i}) = 4; \quad 1 \leq i \leq \left\lfloor \frac{n-1}{3} \right\rfloor$$

$$f(u_{2i-1}) = 9; \quad 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$$

$$f(u_{2i}) = 8; \quad 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$$

Claim:

The  $L(2,1)$  labeling number of a double triangular snake graph  $D(T_n) = 9$ .

Case(i):

Let  $x, y$  be any two vertices in  $V_1(G)$ .

Clearly  $x$  and  $y$  are non-adjacent vertices on  $V_1(G)$ , such that  $x = v_{2i-1}$  and  $y = v_{2i}$ .

Then  $f(x) = 6$ ,  $f(y) = 7$  and  $d(x, y) \geq 2$ .

Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |6-7| \geq 3$ .

Case(ii):

Let  $x, y$  be any two vertices in  $V_2(G)$ .

Subcase (i).

Let  $x, y$  be any two adjacent vertices on  $V_2(G)$ , such that  $x = w_{3i-2}$  and  $y = w_{3i-1}$ .

Then  $f(x) = 0$ ,  $f(y) = 2$  and  $d(x, y) = 1$ .

Therefore  $d(x, y) + |f(x) - f(y)| = 1 + |0-2| = 3$ .

Subcase (ii).

Let  $x, y$  be any two non-adjacent vertices on  $V_2(G)$ , such that  $x = w_{3i-2}$  and  $y = w_{3i}$ . Then  $f(x) = 0$ ,  $f(y) = 4$  and  $d(x, y) \geq 2$ . Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |0-4| \geq 3$ .

Case(iii):

Let  $x, y$  be any two vertices in  $V_3(G)$ .

Clearly  $x$  and  $y$  are non-adjacent vertices on  $V_3(G)$ , such that  $x = u_{2i-1}$  and  $y = u_{2i}$ . Then  $f(x) = 9$ ,  $f(y) = 8$  and  $d(x, y) \geq 2$ . Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |9-8| \geq 3$ .

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Case(iv):

Let  $x, y$  be any two vertices, on the  $V_1(G)$  and  $V_2(G)$  respectively

Subcase (i).

Let  $x, y$  be any two adjacent vertices, on  $V_1(G)$  and  $V_2(G)$  respectively. Let  $x$  be a vertex on  $V_1(G)$  and  $y$  be a vertex on  $V_2(G)$  such that  $x = v_{2i-1}$  and  $y = w_{3i-2}$ . Then  $f(x) = 6$  and  $f(y) = 0$  and  $d(x, y) = 1$ . Therefore  $d(x, y) + |f(x) - f(y)| = 1 + |6-0| \geq 3$ .

Subcase (ii).

Let  $x, y$  be any two non-adjacent vertices, on  $V_1(G)$  and  $V_2(G)$  respectively.

Let  $x$  be a vertex on  $V_1(G)$  and  $y$  be a vertex on  $V_2(G)$  such that  $x = v_{2i-1}$  and  $y = w_{3i}$ . Then  $f(x) = 6$  and  $f(y) = 4$ , then  $d(x, y) \geq 2$ . Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |6-4| \geq 3$ .

Case(v):

Let  $x, y$  be any two vertices, on the  $V_2(G)$  and  $V_3(G)$  respectively

Subcase (i).

Let  $x, y$  be any two adjacent vertices, on  $V_2(G)$  and  $V_3(G)$  respectively. Let  $x$  be a vertex on  $V_2(G)$  and  $y$  be a vertex on  $V_3(G)$  such that  $x = w_{3i-2}$  and  $y = u_{2i-1}$ . Then  $f(x) = 0$  and  $f(y) = 9$  and  $d(x, y) = 1$ . Therefore  $d(x, y) + |f(x) - f(y)| = 1 + |0-9| \geq 3$ .

Subcase (ii).

Let  $x, y$  be any two non-adjacent vertices, on  $V_2(G)$  and  $V_3(G)$  respectively.

Let  $x$  be a vertex on  $V_2(G)$  and  $y$  be a vertex on  $V_3(G)$  such that  $x = w_{3i-2}$  and  $y = u_{2i}$ . Then  $f(x) = 0$  and  $f(y) = 8$ , then  $d(x, y) \geq 2$ . Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |0-8| \geq 3$ .

Case(vi):

Let  $x, y$  be any two vertices in  $V_1(G)$  and  $V_3(G)$  respectively. Clearly  $x$  and  $y$  are non-adjacent vertices. such that  $x = v_{2i-1}$  and  $y = u_{2i}$ . Then  $f(x) = 6$ ,  $f(y) = 8$  and  $d(x, y) \geq 2$ .

Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |6-8| \geq 3$ .

Similarly, for all the other possibilities of  $x$  and  $y$  we find that  $d(x, y) + |f(x) - f(y)| \geq 3$ . Therefore the  $L(2,1)$  labeling number of  $D(T_n)$  is  $\lambda(G) = 9$ .

Example 3.3

The  $L(2,1)$  labeling of a double triangular snake  $D(T_6)$  is shown in Figure 3.3

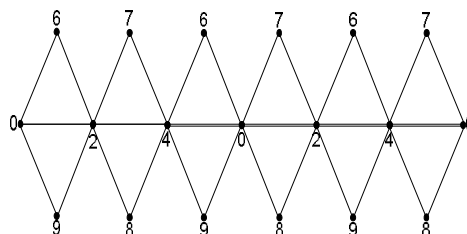


Figure 3.3

Hence the  $L(2,1)$  labeling of a double triangular snake  $D(T_n)$  is  $\lambda(G) = 9$ .

Theorem 3.4.

The  $L(2,1)$  labeling number  $\lambda(G)$  of an alternate double triangular snake graph  $A(D(T_n))$  is 7, for  $n \geq 2$  and  $n \equiv 0 \pmod{2}$ .

Proof.

Denote the vertices of an alternate double triangular snake graph  $A(D(T_n))$  as follows:

$$V(G) = V_1 \cup V_2 \cup V_3 \text{ where } V_1 = \{v_i / 1 \leq i \leq \frac{n}{2}\}, V_2 = \{w_i / 1 \leq i \leq n\}, V_3 = \{u_i / 1 \leq i \leq \frac{n}{2}\}$$

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Define a mapping  $f : V(G) \rightarrow N \cup \{0\}$  by

$$f(v_i) = 0; \quad 1 \leq i \leq \frac{n}{2}$$

$$f(w_{3i-2}) = 3; \quad 1 \leq i \leq \left\lceil \frac{n}{3} \right\rceil \quad f(w_{3i-1}) = 5; \quad 1 \leq i \leq \left\lceil \frac{n-1}{3} \right\rceil$$

$$f(w_{3i}) = 7; \quad 1 \leq i \leq \left\lceil \frac{n-1}{3} \right\rceil$$

$$f(u_i) = 1; \quad 1 \leq i \leq \frac{n}{2}$$

Case(i):

Let  $x, y$  be any two vertices in  $V_1(G)$ .

Clearly  $d(x, y) \geq 3$ . Hence the condition for  $L(2, 1)$  labeling is satisfied.

Case(ii):

Let  $x, y$  be any two vertices in  $V_2(G)$ .

Subcase (i).

Let  $x, y$  be any two adjacent vertices on  $V_2(G)$ , such that  $x = w_{3i-2}$  and  $y = w_{3i-1}$ .

Then  $f(x) = 3, f(y) = 5$  and  $d(x, y) = 1$ .

Therefore  $d(x, y) + |f(x) - f(y)| = 1 + |3-5| = 3$ .

Subcase (ii).

Let  $x, y$  be any two non-adjacent vertices on  $V_2(G)$ , such that  $x = w_{3i-2}$  and  $y = w_{3i}$ . Then  $f(x) = 3, f(y) = 7$  and  $d(x, y) \geq 2$ . Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |3-7| \geq 3$ .

Case(iii):

Let  $x, y$  be any two vertices in  $V_3(G)$ .

Clearly  $d(x, y) \geq 3$ . Hence the condition for  $L(2, 1)$  labeling is satisfied.

Case(iv):

Let  $x, y$  be any two vertices, on the  $V_1(G)$  and  $V_2(G)$  respectively

Subcase (i).

Let  $x, y$  be any two adjacent vertices, on  $V_1(G)$  and  $V_2(G)$  respectively. Let  $x$  be a vertex on  $V_1(G)$  and  $y$  be a vertex on  $V_2(G)$  such that  $x = v_i$  and  $y = w_{3i-2}$ . Then  $f(x) = 0$  and  $f(y) = 3$  and  $d(x, y) = 1$ . Therefore  $d(x, y) + |f(x) - f(y)| = 1 + |0-3| \geq 3$ .

Subcase (ii).

Let  $x, y$  be any two non-adjacent vertices, on  $V_1(G)$  and  $V_2(G)$  respectively.

Let  $x$  be a vertex on  $V_1(G)$  and  $y$  be a vertex on  $V_2(G)$  such that  $x = v_i$  and  $y = w_{3i}$ . Then  $f(x) = 0$  and  $f(y) = 7$ , then  $d(x, y) \geq 2$ . Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |0-7| \geq 3$ .

Case(v):

Let  $x, y$  be any two vertices, on the  $V_2(G)$  and  $V_3(G)$  respectively

Subcase (i).

Let  $x, y$  be any two adjacent vertices, on  $V_2(G)$  and  $V_3(G)$  respectively. Let  $x$  be a vertex on  $V_2(G)$  and  $y$  be a vertex on  $V_3(G)$  such that  $x = w_{3i-2}$  and  $y = u_i$ . Then  $f(x) = 3$  and  $f(y) = 1$  and  $d(x, y) = 1$ . Therefore  $d(x, y) + |f(x) - f(y)| = 1 + |3-1| \geq 3$ .

Subcase (ii).

Let  $x, y$  be any two non-adjacent vertices, on  $V_2(G)$  and  $V_3(G)$  respectively.

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Let  $x$  be a vertex on  $V_2(G)$  and  $y$  be a vertex on  $V_3(G)$  such that  $x = w_{3i}$  and  $y = u_i$ . Then  $f(x)=7$  and  $f(y) = 1$ , then  $d(x, y) \geq 2$ . Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |7-1| \geq 3$ .

Case(vi):

Let  $x, y$  be any two vertices in  $V_1(G)$  and  $V_3(G)$  respectively. Clearly  $x$  and  $y$  are non-adjacent vertices. such that  $x = v_i$  and  $y = u_i$ . then  $f(x) = 0$ ,  $f(y) = 1$  and  $d(x, y) \geq 2$ .

Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |0-1| = 3$ .

Similarly, for all the other possibilities of  $x$  and  $y$  we find that  $d(x, y) + |f(x) - f(y)| \geq 3$ . Therefore the  $L(2,1)$  labeling number of  $A(D(T_n))$  is  $\lambda(G) = 7$ .

Example 3.4  $L(2,1)$  labeling of alternate double triangular snake graph  $A(D(T_4))$  is shown in Figure 3.4

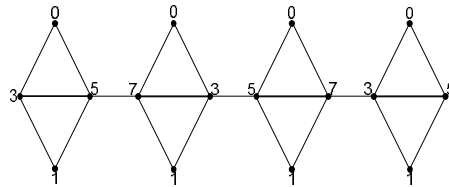


Figure 3.4

Hence the  $L(2,1)$  labeling number  $\lambda(G)$  of an alternate double triangular snake graph  $A(D(T_n))$  is 7.

Theorem 3.5.

The  $L(2,1)$  labeling number  $\lambda(G)$  of a triple triangular snake  $T(T_n)$  is 11, for  $n \geq 3$ .

Proof.

Denote the vertices of a triple triangular snake graph as follows:

$V(G) = V_1 \cup V_2 \cup V_3 \cup V_4$  where  $V_1 = \{v_i / 1 \leq i \leq n-1\}$ ,  $V_2 = \{w_i / 1 \leq i \leq n\}$ ,  $V_3 = \{u_i / 1 \leq i \leq n-1\}$ ,  $V_4 = \{t_i / 1 \leq i \leq n-1\}$

Define a mapping  $f: V(G) \rightarrow \mathbb{N} \cup \{0\}$  by

$$f(v_{2i-1}) = 6; \quad 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$$

$$f(v_{2i}) = 7; \quad 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$$

$$f(w_{3i-2}) = 0; \quad 1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor \quad f(w_{3i-1}) = 2; \quad 1 \leq i \leq \left\lfloor \frac{n-1}{3} \right\rfloor$$

$$f(w_{3i}) = 4; \quad 1 \leq i \leq \left\lfloor \frac{n-1}{3} \right\rfloor$$

$$f(u_{2i-1}) = 9; \quad 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$$

$$f(u_{2i}) = 8; \quad 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$$

$$f(t_{2i-1}) = 10; \quad 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$$



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$$f(t_{2i}) = 11; \quad 1 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor$$

Claim:

The  $L(2,1)$  labeling number of a triple triangular snake graph  $T(T_n) = 11$ .

Case(i):

Let  $x, y$  be any two vertices in  $V_1(G)$ .

Clearly  $x$  and  $y$  are non-adjacent vertices on  $V_1(G)$ , such that  $x = v_{2i-1}$  and  $y = v_{2i}$ .

Then  $f(x) = 6$ ,  $f(y) = 7$  and  $d(x, y) \geq 2$ .

Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |6-7| \geq 3$ .

Case(ii):

Let  $x, y$  be any two vertices in  $V_2(G)$ .

Subcase (i).

Let  $x, y$  be any two adjacent vertices on  $V_2(G)$ , such that  $x = w_{3i-2}$  and  $y = w_{3i-1}$ .

Then  $f(x) = 0$ ,  $f(y) = 2$  and  $d(x, y) = 1$ .

Therefore  $d(x, y) + |f(x) - f(y)| = 1 + |0-2| = 3$ .

Subcase (ii).

Let  $x, y$  be any two non-adjacent vertices on  $V_2(G)$ , such that  $x = w_{3i-2}$  and  $y = w_{3i}$ . Then  $f(x) = 0$ ,  $f(y) = 4$  and  $d(x, y) \geq 2$ . Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |0-4| \geq 3$ .

Case(iii):

Let  $x, y$  be any two vertices in  $V_3(G)$ .

Clearly  $x$  and  $y$  are non-adjacent vertices on  $V_3(G)$ , such that  $x = u_{2i-1}$  and  $y = u_{2i}$ . Then  $f(x) = 9$ ,  $f(y) = 8$  and  $d(x, y) \geq 2$ . Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |9-8| \geq 3$ .

Case(iv):

Let  $x, y$  be any two vertices in  $V_4(G)$ .

Clearly  $x$  and  $y$  are non-adjacent vertices on  $V_3(G)$ , such that  $x = u_{2i-1}$  and  $y = u_{2i}$ . Then  $f(x) = 10$ ,  $f(y) = 11$  and  $d(x, y) \geq 2$ . Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |10-11| \geq 3$ .

Case(v):

Let  $x, y$  be any two vertices, on the  $V_1(G)$  and  $V_2(G)$  respectively

Subcase (i).

Let  $x, y$  be any two adjacent vertices, on  $V_1(G)$  and  $V_2(G)$  respectively. Let  $x$  be a vertex on  $V_1(G)$  and  $y$  be a vertex on  $V_2(G)$  such that  $x = v_{2i-1}$  and  $y = w_{3i-2}$ . Then  $f(x) = 6$  and  $f(y) = 0$  and  $d(x, y) = 1$ . Therefore  $d(x, y) + |f(x) - f(y)| = 1 + |6-0| \geq 3$ .

Subcase (ii).

Let  $x, y$  be any two non-adjacent vertices, on  $V_1(G)$  and  $V_2(G)$  respectively.

Let  $x$  be a vertex on  $V_1(G)$  and  $y$  be a vertex on  $V_2(G)$  such that  $x = v_{2i-1}$  and  $y = w_{3i}$ . Then  $f(x) = 6$  and  $f(y) = 4$ , then  $d(x, y) \geq 2$ . Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |6-4| \geq 3$ .

Case(vi):

Let  $x, y$  be any two vertices, on the  $V_2(G)$  and  $V_3(G)$  respectively

Subcase (i).

Let  $x, y$  be any two adjacent vertices, on  $V_2(G)$  and  $V_3(G)$  respectively. Let  $x$  be a vertex on  $V_2(G)$  and  $y$  be a vertex on  $V_3(G)$  such that  $x = w_{3i-2}$  and  $y = u_{2i-1}$ . Then  $f(x) = 0$  and  $f(y) = 9$  and  $d(x, y) = 1$ . Therefore  $d(x, y) + |f(x) - f(y)| = 1 + |0-9| \geq 3$ .

Subcase (ii).

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Let  $x, y$  be any two non-adjacent vertices, on  $V_2(G)$  and  $V_3(G)$  respectively.

Let  $x$  be a vertex on  $V_2(G)$  and  $y$  be a vertex on  $V_3(G)$  such that  $x = w_{3i-2}$  and  $y = u_{2i}$ . Then  $f(x) = 0$  and  $f(y) = 8$ , then  $d(x, y) \geq 2$ . Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |0 - 8| \geq 3$ .

Case(vii):

Let  $x, y$  be any two vertices in  $V_1(G)$  and  $V_3(G)$  respectively. Clearly  $x$  and  $y$  are non-adjacent vertices. such that  $x = v_{2i-1}$  and  $y = u_{2i}$ . Then  $f(x) = 6$ ,  $f(y) = 8$  and  $d(x, y) \geq 2$ .

Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |6 - 8| \geq 3$ .

Case(viii):

Let  $x, y$  be any two vertices, on the  $V_2(G)$  and  $V_4(G)$  respectively

Subcase (i).

Let  $x, y$  be any two adjacent vertices, on  $V_2(G)$  and  $V_4(G)$  respectively. Let  $x$  be a vertex on  $V_2(G)$  and  $y$  be a vertex on  $V_4(G)$  such that  $x = w_{3i-2}$  and  $y = t_{2i-1}$ . Then  $f(x) = 0$  and  $f(y) = 10$  and  $d(x, y) = 1$ . Therefore  $d(x, y) + |f(x) - f(y)| = 1 + |0 - 10| \geq 3$ .

Subcase (ii).

Let  $x, y$  be any two non-adjacent vertices, on  $V_2(G)$  and  $V_4(G)$  respectively.

Let  $x$  be a vertex on  $V_2(G)$  and  $y$  be a vertex on  $V_4(G)$  such that  $x = w_{3i-2}$  and  $y = t_{2i}$ . Then  $f(x) = 0$  and  $f(y) = 11$ , then  $d(x, y) \geq 2$ . Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |0 - 11| \geq 3$ .

Case(ix):

Let  $x, y$  be any two vertices in  $V_1(G)$  and  $V_4(G)$  respectively. Clearly  $x$  and  $y$  are non-adjacent vertices. such that  $x = v_{2i-1}$  and  $y = t_{2i}$ . Then  $f(x) = 6$ ,  $f(y) = 11$  and  $d(x, y) \geq 2$ .

Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |6 - 11| \geq 3$ .

Case(x):

Let  $x, y$  be any two vertices in  $V_3(G)$  and  $V_4(G)$  respectively. Clearly  $x$  and  $y$  are non-adjacent vertices. such that  $x = u_{2i-1}$  and  $y = t_{2i}$ . Then  $f(x) = 9$ ,  $f(y) = 11$  and  $d(x, y) \geq 2$ .

Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |9 - 11| \geq 3$ .

Similarly, for all the other possibilities of  $x$  and  $y$  we find that  $d(x, y) + |f(x) - f(y)| \geq 3$ . Therefore the  $L(2,1)$  labeling number of  $T(T_n)$  is  $\lambda(G) = 11$ .

Example 3.5

The  $L(2,1)$  labeling of a triple triangular snake  $T(T_6)$  is shown in Figure 3.5

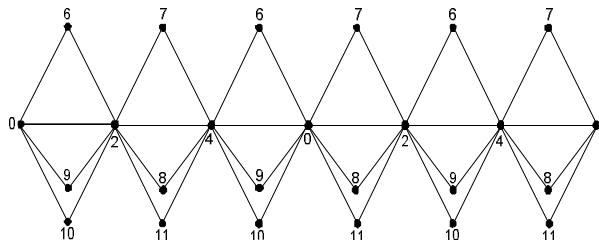


Figure 3.5

Hence the  $L(2,1)$  labeling of a triple triangular snake  $T(T_n)$  is  $\lambda(G) = 11$ .

.Theorem 3.6.

The  $L(2,1)$  labeling number  $\lambda(G)$  of an alternate triple triangular snake graph  $A(T(T_n))$  is 9, for  $n \geq 2$  and  $n \equiv 0 \pmod{2}$ .

Proof.

Denote the vertices of an alternate double triangular snake graph  $A(T(T_n))$  as follows:

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$V(G) = V_1 \cup V_2 \cup V_3 \cup V_4$  where  $V_1 = \{v_i / 1 \leq i \leq \frac{n}{2}\}$ ,  $V_2 = \{w_i / 1 \leq i \leq n\}$ ,  $V_3 = \{u_i / 1 \leq i \leq \frac{n}{2}\}$ ,  $V_4 = \{t_i / 1 \leq i \leq \frac{n}{2}\}$

Define a mapping  $f : V(G) \rightarrow \mathbb{N} \cup \{0\}$  by

$$f(v_i) = 0; \quad 1 \leq i \leq \frac{n}{2}$$

$$f(w_{3i-2}) = 3; \quad 1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor \quad f(w_{3i-1}) = 5; \quad 1 \leq i \leq \left\lfloor \frac{n-1}{3} \right\rfloor$$

$$f(w_{3i}) = 7; \quad 1 \leq i \leq \left\lfloor \frac{n-1}{3} \right\rfloor$$

$$f(u_i) = 1; \quad 1 \leq i \leq \frac{n}{2}$$

$$f(t_i) = 9; \quad 1 \leq i \leq \frac{n}{2}$$

Now let us find the  $L(2,1)$  labeling number of this graph.

Case(i):

Let  $x, y$  be any two vertices in  $V_1(G)$ .

Clearly  $d(x, y) \geq 3$ . Hence the condition for  $L(2,1)$  labeling is satisfied.

Case(ii):

Let  $x, y$  be any two vertices in  $V_2(G)$ .

Subcase (i).

Let  $x, y$  be any two adjacent vertices on  $V_2(G)$ , such that  $x = w_{3i-2}$  and  $y = w_{3i-1}$ .

Then  $f(x) = 3, f(y) = 5$  and  $d(x, y) = 1$ .

Therefore  $d(x, y) + |f(x) - f(y)| = 1 + |3-5| = 3$ .

Subcase (ii).

Let  $x, y$  be any two non-adjacent vertices on  $V_2(G)$ , such that  $x = w_{3i-2}$  and  $y = w_{3i}$ . Then  $f(x) = 3, f(y) = 7$  and  $d(x, y)$

$\geq 2$ . Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |3-7| \geq 3$ .

Case(iii):

Let  $x, y$  be any two vertices in  $V_3(G)$ .

Clearly  $d(x, y) \geq 3$ . Hence the condition for  $L(2,1)$  labeling is satisfied.

Case(iv):

Let  $x, y$  be any two vertices in  $V_4(G)$ .

Clearly  $d(x, y) \geq 3$ . Hence the condition for  $L(2,1)$  labeling is satisfied.

Case(v):

Let  $x, y$  be any two vertices, on the  $V_1(G)$  and  $V_2(G)$  respectively

Subcase (i).

Let  $x, y$  be any two adjacent vertices, on  $V_1(G)$  and  $V_2(G)$  respectively. Let  $x$  be a vertex on  $V_1(G)$  and  $y$  be a vertex on  $V_2(G)$  such that  $x = v_i$  and  $y = w_{3i-2}$ . Then  $f(x) = 0$  and  $f(y) = 3$  and  $d(x, y) = 1$ . Therefore  $d(x, y) + |f(x) - f(y)| = 1 + |0-3| \geq 3$ .

Subcase (ii).

Let  $x, y$  be any two non-adjacent vertices, on  $V_1(G)$  and  $V_2(G)$  respectively.

Let  $x$  be a vertex on  $V_1(G)$  and  $y$  be a vertex on  $V_2(G)$  such that  $x = v_i$  and  $y = w_{3i}$ . Then  $f(x) = 0$  and  $f(y) = 7$ , then  $d(x, y) \geq 2$ . Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |0-7| \geq 3$ .

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Case(vi):

Let  $x, y$  be any two vertices, on the  $V_2(G)$  and  $V_3(G)$  respectively

Subcase (i).

Let  $x, y$  be any two adjacent vertices, on  $V_2(G)$  and  $V_3(G)$  respectively. Let  $x$  be a vertex on  $V_2(G)$  and  $y$  be a vertex on  $V_3(G)$  such that  $x = w_{3i-2}$  and  $y = u_i$ . Then  $f(x) = 3$  and  $f(y) = 1$  and  $d(x, y) = 1$ . Therefore  $d(x, y) + |f(x) - f(y)| = 1 + |3-1| \geq 3$ .

Subcase (ii).

Let  $x, y$  be any two non-adjacent vertices, on  $V_2(G)$  and  $V_3(G)$  respectively.

Let  $x$  be a vertex on  $V_2(G)$  and  $y$  be a vertex on  $V_3(G)$  such that  $x = w_{3i}$  and  $y = u_i$ . Then  $f(x)=7$  and  $f(y) = 1$ , then  $d(x, y) \geq 2$ . Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |7-1| \geq 3$ .

Case(vii):

Let  $x, y$  be any two vertices in  $V_1(G)$  and  $V_3(G)$  respectively. Clearly  $x$  and  $y$  are non-adjacent vertices. such that  $x = v_i$  and  $y = u_i$ . then  $f(x) = 0$ ,  $f(y) = 1$  and  $d(x, y) \geq 2$ .

Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |0-1| = 3$ .

Case(viii):

Let  $x, y$  be any two vertices, on the  $V_2(G)$  and  $V_4(G)$  respectively

Subcase (i).

Let  $x, y$  be any two adjacent vertices, on  $V_2(G)$  and  $V_4(G)$  respectively. Let  $x$  be a vertex on  $V_2(G)$  and  $y$  be a vertex on  $V_4(G)$  such that  $x = w_{3i-2}$  and  $y = t_i$ . Then  $f(x) = 3$  and  $f(y) = 9$  and  $d(x, y) = 1$ . Therefore  $d(x, y) + |f(x) - f(y)| = 1 + |3-9| \geq 3$ .

Subcase (ii).

Let  $x, y$  be any two non-adjacent vertices, on  $V_2(G)$  and  $V_4(G)$  respectively.

Let  $x$  be a vertex on  $V_2(G)$  and  $y$  be a vertex on  $V_4(G)$  such that  $x = w_{3i}$  and  $y = t_i$ . Then  $f(x)=7$  and  $f(y) = 9$ , then  $d(x, y) \geq 2$ . Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |7-9| \geq 3$ .

Case(ix):

Let  $x, y$  be any two vertices in  $V_1(G)$  and  $V_4(G)$  respectively. Clearly  $x$  and  $y$  are non-adjacent vertices. such that  $x = v_i$  and  $y = t_i$ . then  $f(x) = 0$ ,  $f(y) = 9$  and  $d(x, y) \geq 2$ .

Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |0-9| \geq 3$ .

Case(x):

Let  $x, y$  be any two vertices in  $V_3(G)$  and  $V_4(G)$  respectively. Clearly  $x$  and  $y$  are non-adjacent vertices. such that  $x = u_i$  and  $y = t_i$ . then  $f(x) = 1$ ,  $f(y) = 9$  and  $d(x, y) \geq 2$ .

Therefore  $d(x, y) + |f(x) - f(y)| \geq 2 + |1-9| \geq 3$ .

Similarly, for all the other possibilities of  $x$  and  $y$  we find that  $d(x, y) + |f(x) - f(y)| \geq 3$ . Therefore the  $L(2,1)$  labeling number of  $A(T(T_n))$  is  $\lambda(G) = 9$ .

Example 3.6

$L(2,1)$  labeling of an alternate triple triangular snake graph  $A(T(T_4))$  is shown in Figure 3.6

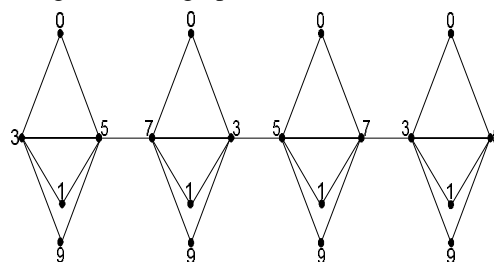


Figure 3.6

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Hence the  $L(2,1)$  labeling number  $\lambda(G)$  of an alternate triple triangular snake graph  $A(T(T_n))$  is 9.

## IV. CONCLUSION

In this paper, the  $L(2, 1)$  labeling of triangular snake, double triangular snake, alternate triangular snake, alternate double triangular snake, triple triangular snake and alternate triple triangular snake have been determined.

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